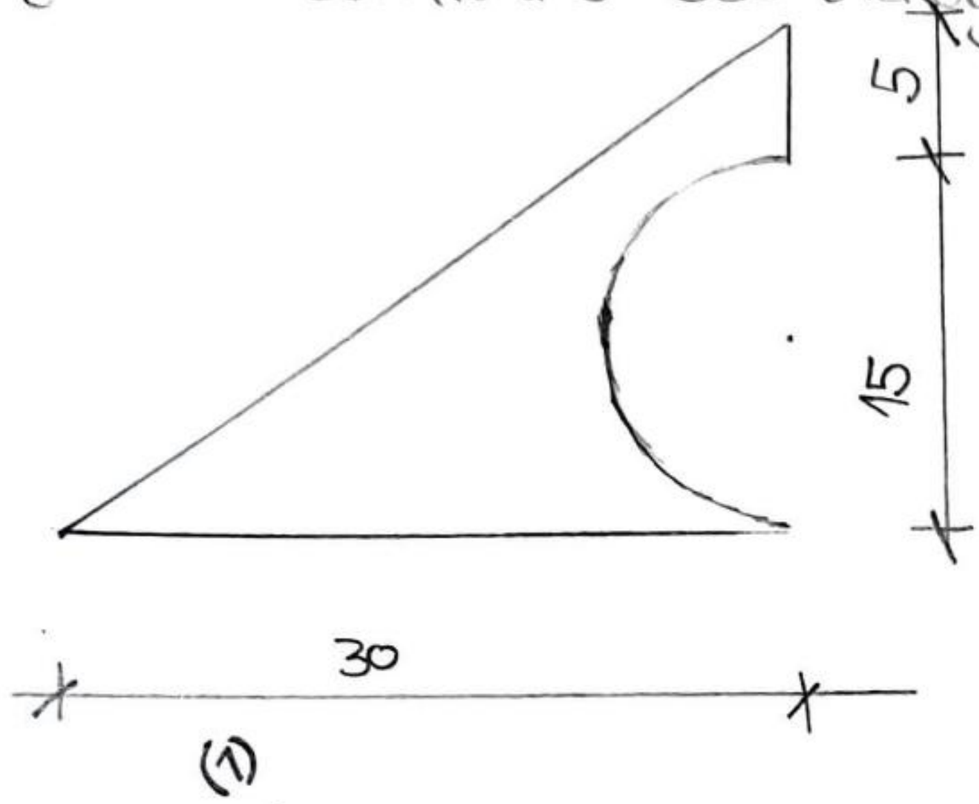
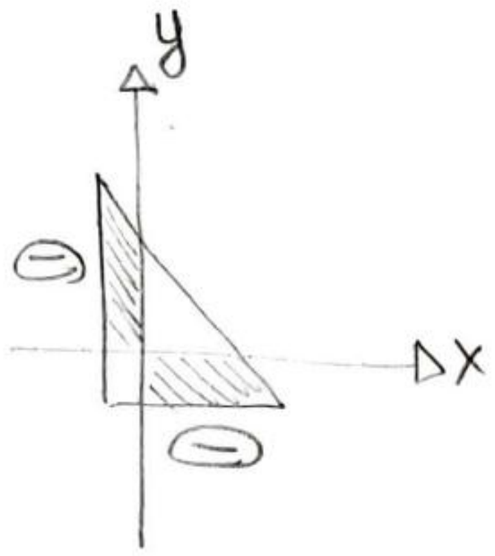
1. Za presjek na slici odrediti glavne centralne momente inercije i glavne centralne ose inercije. Rezultat provjeriti Mohrovim krugom.

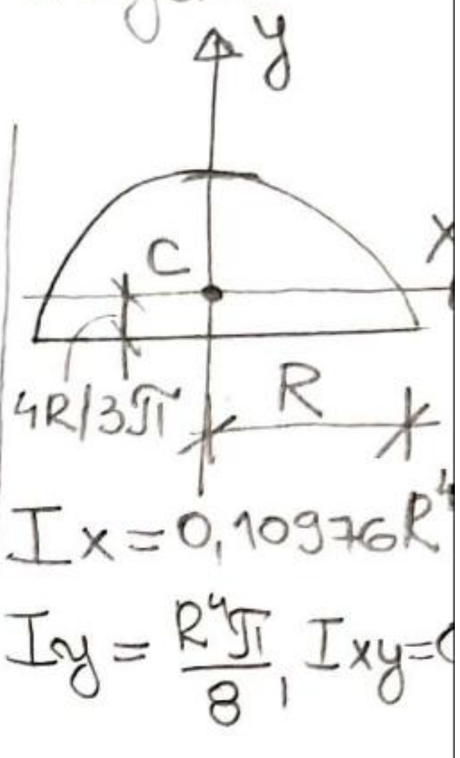




$$I_x = \frac{h^3 b}{36}$$

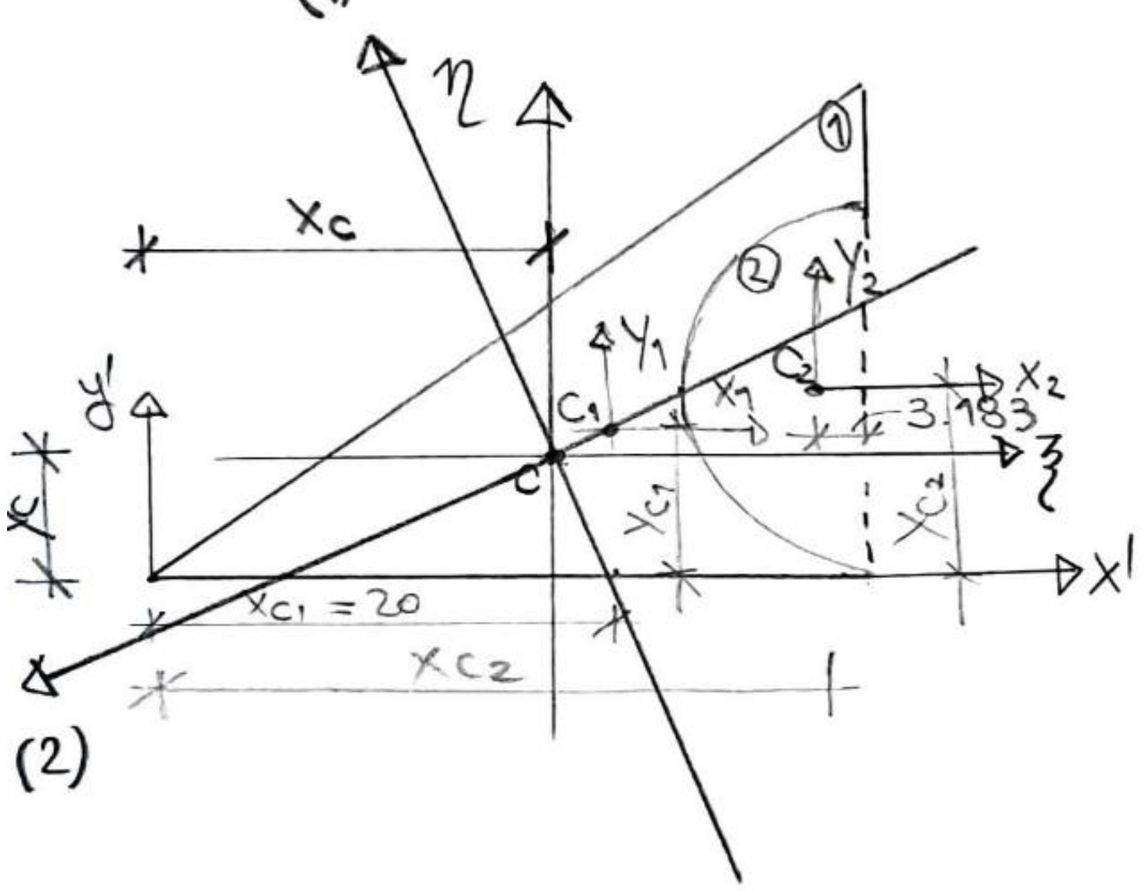
$$I_y = \frac{b^3 h}{36}$$

$$I_{xy} = -\frac{b^2 h^2}{72}$$



$I_x = 0{,}10976R^4$

$I_y = \frac{R^4\pi}{8}$, $I_{xy} = 0$



$C_1(x_{c_1}, y_{c_1})$, $C_1(20; 6{,}667)$

$C_2(x_{c_2}, y_{c_2})$, $C_2(26{,}817; 7{,}5)$

$A_1 = \frac{1}{2} 20 \cdot 30 = 300\,cm^2$

$A_2 = \frac{R^2\pi}{2} = \frac{7{,}5^2\pi}{2} = 88{,}357\,cm^2$

* Odredivanje težišta složene figure:

$$x_c = \frac{x_{c_1} A_1 - x_{c_2} A_2}{A_1 - A_2} = \frac{20 \cdot 300 - 26{,}817 \cdot 88{,}357}{300 - 88{,}357} = 17{,}154\,cm$$

$$y_c = \frac{y_{c_1} A_1 - y_{c_2} A_2}{A_1 - A_2} = \frac{6{,}667 \cdot 300 - 7{,}5 \cdot 88{,}357}{300 - 88{,}357} = 6{,}319\,cm$$

$C(17{,}154; 6{,}319)$

* Momenti inercije:

$I_\xi = I_{\xi_1} - I_{\xi_2}$

$I_\xi^{①} = I_x^{①} + (y_{c_1} - y_c)^2 \cdot A_1 = \frac{20^3 \cdot 30}{36} + (\underbrace{6{,}667 - 6{,}319}_{0{,}348})^2 \cdot 300 = 6703\,cm^4$

$I_\xi^{②} = I_x^{②} + (y_{c_2} - y_c)^2 \cdot A_2 = \frac{7{,}5^4\pi}{8} + (\underbrace{7{,}5 - 6{,}319}_{1{,}181})^2 \cdot 88{,}357 = 1365{,}761\,cm^4$

$I_\xi = I_\xi^{①} - I_\xi^{②} = 5337{,}239\,cm^4$

$I_\eta = I_\eta^{①} - I_\eta^{②}$

$I_\eta^{①} = I_y^{①} + (x_{c_1} - x_c)^2 \cdot A_1 = \frac{30^3 \cdot 20}{36} + (\underbrace{20 - 17{,}154}_{2{,}846})^2 \cdot 300 = 17429{,}915\,cm^4$

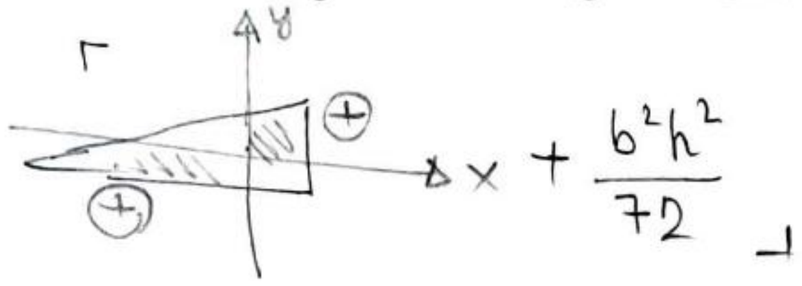
$I_\eta^{②} = I_y^{②} + (x_{c_2} - x_c)^2 \cdot A_2 = 0{,}10976 \cdot 7{,}5^4 + (\underbrace{26{,}817 - 17{,}154}_{9{,}663})^2 \cdot 88357 = 8597{,}496\,cm^4$

$$I_\eta = I_{\eta①} - I_\eta② = 8832,419 cm^4$$

* Centrifugalni moment inercije:

$$I_{\xi\eta} = I_{\xi\eta}① - I_{\xi\eta}②$$

$$I_{\xi\eta}① = I_{xy}① + (y_{c_1} - y_c)\cdot(x_{c_1} - x_c)\cdot A = + \frac{20^2\cdot 30^2}{72} + 0,348\cdot 2,846\cdot 300 = 5297,122\ cm^4$$



$$I_{\xi\eta}② = I_{xy}② + (y_{c_2} - y_c)\cdot(x_{c_2} - x_c)\cdot A = 0 + 1,131\cdot 9,663\cdot 88,357 = 1008,330\ cm^4$$

$$I_{\xi\eta} = 4288,792 cm^4$$

* Glavni centralni momenti inercije:

$$I_{1/2} = \frac{1}{2}(I_\xi + I_\eta) \pm 0,5\cdot\sqrt{(I_\xi - I_\eta)^2 + 4 I_{\xi\eta}^2} = 7084,829 \pm 4631,178$$

$$I_1 = 11716,007 cm^4$$

$$I_2 = 2453,651 cm^4$$

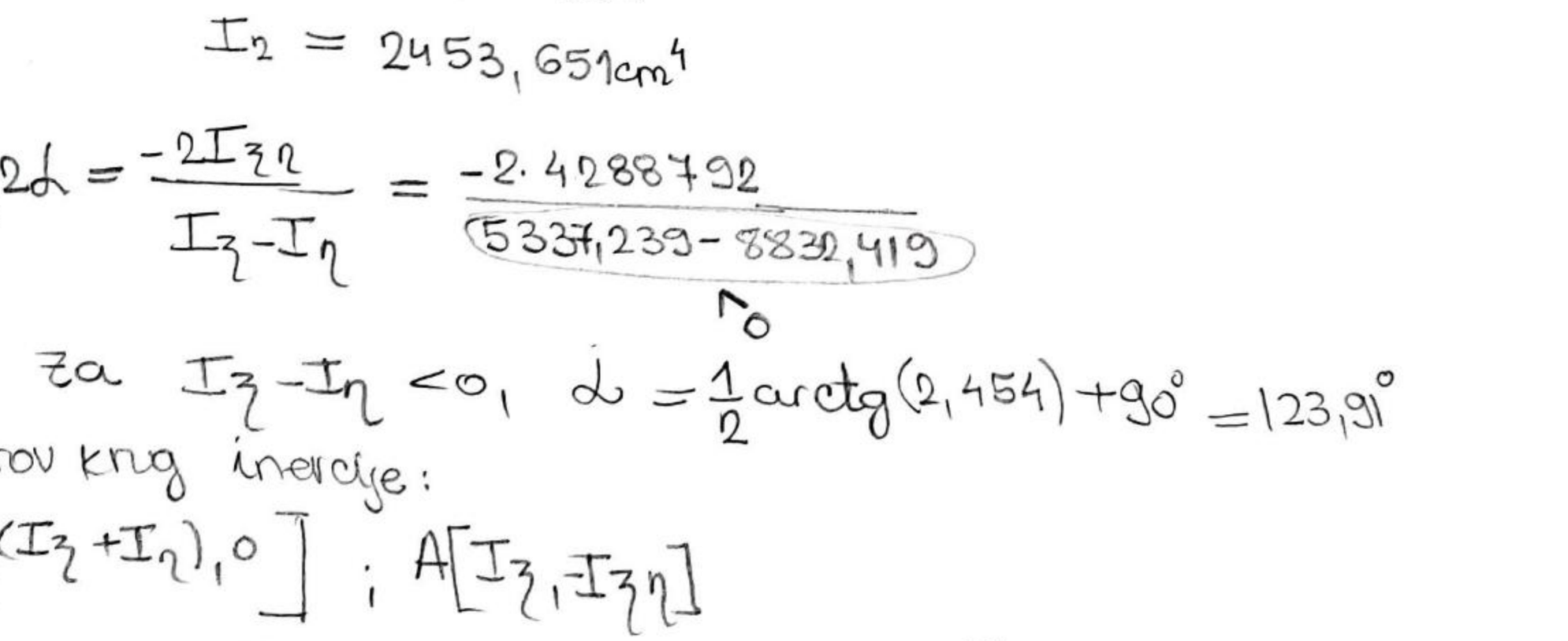
$$tg2\alpha = \frac{-2I_{\xi\eta}}{I_\xi - I_\eta} = \frac{-2\cdot 4288792}{(5337,239 - 8832,419)} \quad (<0)$$

za $I_\xi - I_\eta < 0$, $\alpha = \frac{1}{2}arctg(2,454) + 90° = 123,91°$

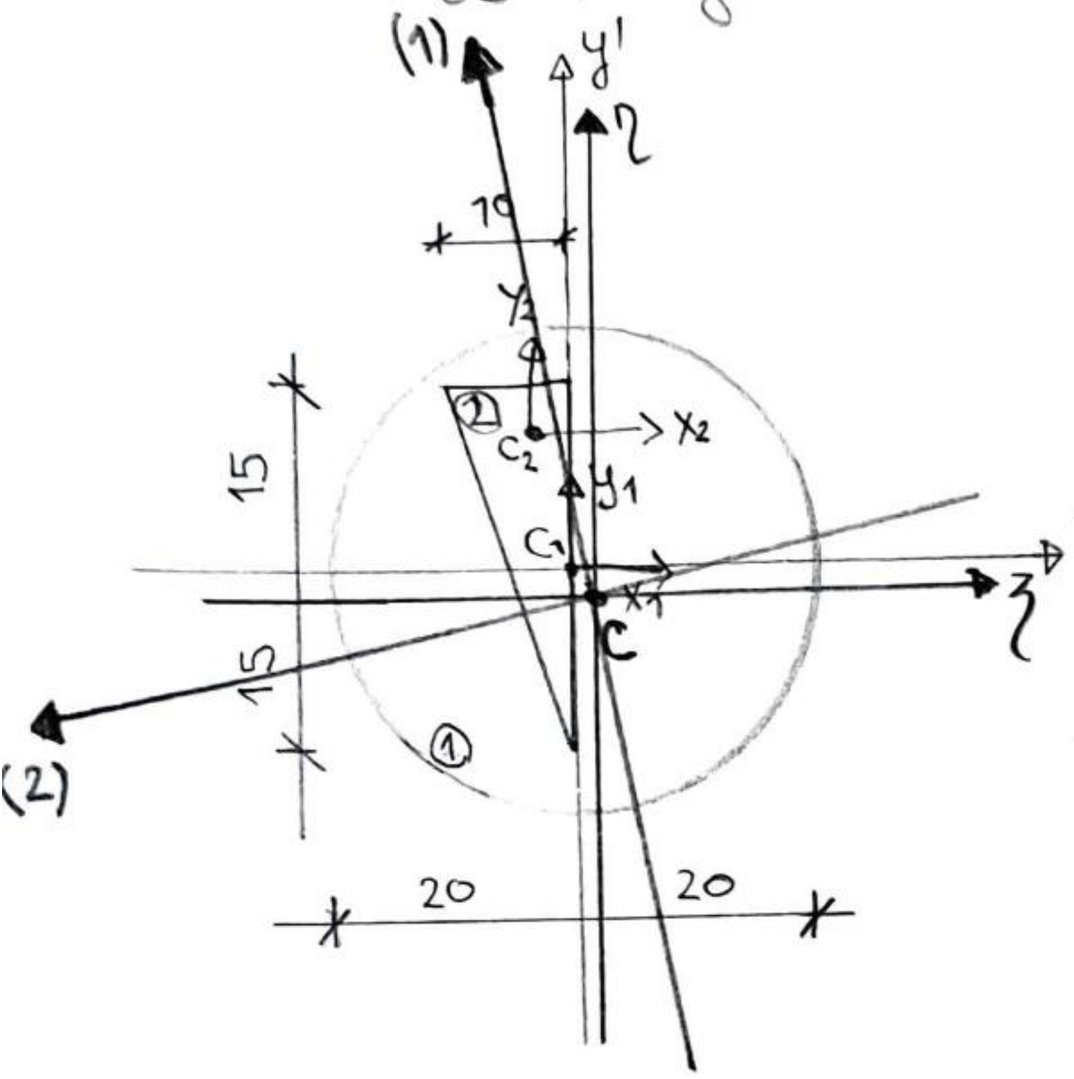
* Morov krug inercije:

$C\left[\frac{1}{2}(I_\xi + I_\eta), 0\right]$; $A[I_\xi, -I_{\xi\eta}]$

$C(7084,82; 0]$; $A[5337,239; -4288,792]$; $1cm = 2000 cm^4$



2. Za presjek na slici odrediti glavne centralne momente inercije i glavne centralne ose inercije. Rezultat provjeriti Mohrom krugom.



$C_1(x_{c_1}, y_{c_1})$, $C_1(0,0)$ $A_1 = R^2\pi = 1256{,}637\ cm^2$

$C_2(x_{c_2}, y_{c_2})$; $C_2(-3{,}3\dot{3}\ ;\ 5)$; $A_2 = \frac{1}{2}\,30\cdot 10$

$A_2 = 150\ cm^2$

*odredivanje težišta složene figure

$$x_c = \frac{0 + 3{,}3\dot{3}\cdot 150}{1256{,}637 - 150} = 0{,}4518cm$$

$$y_c = \frac{0 - 5\cdot 150}{1106{,}637} = -0{,}6778cm$$

$C(0{,}4518; -0{,}6778)$

* Momenti inercije:

$I_\zeta = I_{\zeta①} - I_{\zeta②}$

$I_\zeta^① = \frac{20^4\cdot\pi}{4} + (0+0{,}6778)^2\cdot 1256{,}637 = 126241{,}0211cm^4$

$I_\zeta^② = \frac{30^3\cdot 10}{36} + (5+0{,}6778)^2\cdot 150 = 12335{,}612cm^4$

$I_\zeta = 113905{,}4092cm^4$

$I_\eta = I_\eta^① - I_\eta^②$

$I_\eta^① = \frac{20^4\cdot\pi}{4} + (0-0{,}4518)^2\cdot 1256{,}637 = 125920{,}215cm^4$

$-3{,}7851$

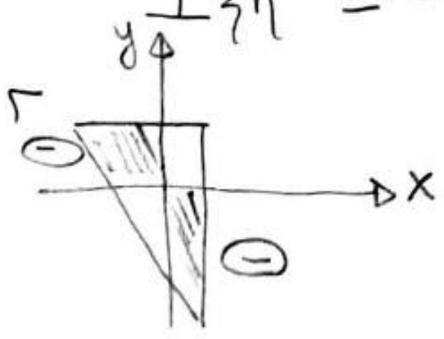
$I_\eta^② = \frac{10^3\cdot 30}{36} + (-3{,}3\dot{3}-0{,}4518)^2\cdot 150 = 2982{,}38cm^4$

$I_\eta = 122937{,}834cm^4$

$I_{\zeta\eta} = I_{\zeta\eta}^① - I_{\zeta\eta}^②$

$I_{\zeta\eta}^① = 0 + 0{,}6778\cdot(-0{,}4518)\cdot 1256{,}637 = -384{,}82cm^4$

$I_{\zeta\eta}^② = -\frac{10^2\cdot 30^2}{72} + 5{,}6778\cdot(-3{,}7851)\cdot 150 = -4473{,}656\ cm^4$



$I_{\zeta\eta} = 4088{,}836cm^4$

Glavni centralni momenti inercije:

$$I_{1/2} = \frac{1}{2}(I_\zeta + I_\eta) \pm 0{,}5 \cdot \sqrt{(I_\zeta - I_\eta)^2 + 4I_{\zeta\eta}^2} = 118421{,}6216 \pm 6092{,}188 \Rightarrow$$

$$I_1 = 124513{,}8096\,cm^4$$

$$I_2 = 112329{,}4336\,cm^4$$

kontrola: $I_1 + I_2 = I_\zeta + I_\eta \Rightarrow 236843{,}24\,cm^4 = 236843{,}24\,cm^4$

Glavne centralne ose inercije:

$$tg2\alpha = \frac{-2I_{\zeta\eta}}{I_\zeta - I_\eta} = \frac{-2 \cdot 4088{,}836}{(113905{,}4092 - 122937{,}834)} = 0{,}90537$$

$$-9032{,}4248$$

za $I_\zeta - I_\eta < 0$, $\alpha = \frac{1}{2} arctg(0{,}90537) + 90° = 111{,}0783°$

Mohr krug:

$C[118421{,}62 ; 0]$; $A[113905{,}4092 ; -4088{,}836]$

$1cm = 7500\,cm^4$